These lecture notes are essential reading for anyone interested in orthogonal polynomials, weighted approximation, and potential theory. It is an achievement of lasting value, possibly as significant as the Pollard–Mergelyan–Akhiezer solutions to Bernstein's approximation problem in the 1950s.

DORON LUBINSKY

TOM H. KOORNWINDER (Ed.), *Wavelets: An Elementary Treatment of Theory and Applications*, Series in Approximations and Decompositions 1, World Scientific, Singapore, 1993, xii + 225 pp.

Based on a four-day intensive course given at the CWI, Amsterdam, this collection of papers is a contribution to help a rather broad audience with understanding wavelets. It aims at giving the general ideas of the basic theory, as well as of some important applications, at a level which is adequate for a mixed audience. The need of such a presentation is evident, although (and since) the wavelet literature is tremendously increasing. Everybody who has ever taught a mathematical course in this direction will appreciate having such a guideline, and anyone who applies (discrete) wavelets to analyze and to process data—wherever these may originate from—will welcome this elementary treatment.

The book consists of twelve articles written mainly by scientists from the CWI and the University of Amsterdam. First there are two introductory expositions. *Wavelets: First Steps*, by N. M. Temme, gives an overview of the continuous and the discrete wavelet transform, as well as of multiresolution analysis. It also shows the general principle how to find functions ϕ satisfying a dilation (or scaling) equation and how to construct the corresponding wavelet in case the integer translates of ϕ are orthonormal. *Wavelets: Mathematical Preliminaries*, by P. W. Hemker *et al.*, compiles some mathematical background including Hilbert space notions, Fourier approximation, but also Riesz bases and frames in Hilbert spaces.

Next follow two articles describing the theory of wavelet analysis. *The Continuous Wavelet Transform*, by T. H. Koornwinder, starts with the transformation formula and its corresponding Parseval identity and inversion formula. Localization in the time-frequency domain is discussed, and an exposition is given on how to deal with the continuous wavelet transform from a more abstract point of view referring to unitary representations of locally compact groups. *Discrete Wavelets and Multiresolution Analysis*, by H. J. A. M. Heijmans, gives the construction of a nested sequence of multiresolution spaces by defining the Fourier transform of the basis function in terms of the usual infinite product. Then the corresponding wavelet basis of $L^2(\mathbb{R})$ is constructed, and the recursive formulas for finding the Fourier-wavelet expansion are given. As an example, the Meyer wavelet is shown to satisfy this construction principle.

The following two contributions *Image Compression Using Wavelets*, by P. Nacken, and *Computing with Daubechies' Wavelets*, by A. B. Olde Daalhuis, show the details of the wavelet decomposition and reconstruction formulas in terms of discrete convolution and down-sampling (upsampling, respectively) procedures. The idea of compression based on the wavelet expansion as well as the corresponding reconstruction error is discussed (but not worked out in detail).

Wavelet Bases Adapted to Inhomogeneous Cases, by P. W. Hemker and F. Plantevin, gives some extensions of the wavelet ideas to cases where typical ingredients of wavelet theory are not present. In particular, they discuss the construction of orthonormal (wavelet) bases for L^2-functions on a bounded domain $\Omega \subset \mathbb{R}^n$ satisfying homogeneous Dirichlet boundary conditions, they introduce wavelets associated with the boundary value problem $D^*a\, Du = f$ on $I = [0, 1]$, where $u(0) = u(1) = 0$, with $D = -i(d/dx)$ and a a (bounded and accretive) complex-valued function, and they deal with wavelets on certain irregular meshes.

The remaining five articles are more directed into giving concrete ideas and hints for applications. *Conjugate Quadrature Filters for Multiresolution Analysis and Synthesis*, by

E. H. Dooijes, explains the cascade-like wavelet algorithms from the viewpoint of two-channel and multi-channel subband coding schemes. *Calculation of the Wavelet Decomposition using Quadrature Formulae*, by W. Sweldens and R. Piessens, gives another introduction into multiresolution approximation, and it shows in detail how to approximate inner products of $f \in L^2(\mathbb{R})$ with wavelets and scaling functions by combinations of point evaluations of f. *Fast Wavelet Transforms and Calderón–Zygmund Operators*, by T. H. Koornwinder, applies periodized wavelets to derive wavelet expansions of certain integral operators. *The Finite Wavelet Transform with an Application to Seismic Processing*, by J. A. H. Alkemade, and *Wavelets Understand Fractals*, by M. Hazewinkel, conclude the book with two further recent applications of wavelet analysis.

The book can be highly recommended to everyone who needs a quick account of wavelet theory as well as some ideas of wavelet applications. Results and basic theorems are stated in a rigorous and very satisfactory way, without overloading the treatment by including too many concisely worked-out proofs. Those interested in a more complete treatment will find enough hints where to look up the details. While not being a textbook for students at an intermediate level, it can be useful as an aide in more advanced courses or seminars. For specialists in the field, the book can serve as a nice reference work; engineers and other people interested in algorithms for the fast wavelet transform will find it a useful guide to go directly to their specific interests. I am convinced that this "elementary treatment of theory and applications" will become a standard reference for a broad audience.

KURT JETTER

S. B. YAKUBOVICH AND YU. F. LUCHKO, *The Hypergeometric Approach to Integral Transforms and Convolutions*, Mathematics and Its Applications **287**, Kluwer Academic, Dordrecht, 1994, xi + 324 pp.

Throughout the history of applied mathematics, investigators have found integral transforms to be of great use in solving practical problems. An integral transform is a mapping $f \mapsto g$ defined by

$$g(y) = \int_0^\infty k(x, y)\, f(x)\, dx,$$

where $k(x, y)$ is called the kernel of the transform. It was observed that many integral transforms possess an inversion formula of the type

$$f(x) = \int_0^\infty k^*(x, y)\, g(y)\, dy.$$

(Sometimes the path of integration in the second integral is complex.) The two formulas above constitute an integral transform pair, usually named after the inventor.

The authors of this treatise study two sorts of transforms. The first they call Mellin convolution type transforms. Among the best known of these are $k(x, y) = e^{-xy}$ (Laplace), $k(x, y) = x^{y-1}$ (Mellin), $k(x, y) = \cos xy$ (Fourier cosine), $k(x, y) = \sqrt{xy}\ K_\nu(xy)$ (Meijer), $k(x, y) = (xy)^{\mu - 1/2} e^{-xy/2} W_{\kappa,\mu}(xy)$ (generalized Meijer), and $k(x, y) = (x + y)^{-\rho}$ (Stieltjes). The reader will observe that all these kernels are hypergeometric type functions—indeed, any other sort of kernel is virtually unknown. The inversion formulas for these transforms are known and their kernels are closely related hypergeometric functions. Further, integration in both the transform and its inversion formula are conducted with respect to the argument of the hypergeometric function.